Uncertain Timestamp Model

Clas Veibäck, Gustaf Hendeby and Fredrik Gustafsson (firstname.lastname@liu.se)

Problem Formulation

Traditionally in estimation, the information in measurements is assumed to be noisy, while the sampling times are assumed to be accurately known. However, there are many applications where observations are also uncertain in time.

Uncertain Timestamp Model

A traditional linear Gaussian state-space model is considered, with $\mathcal{K} = \{1, \dots, N\}$,

$$
\begin{aligned}
\mathbf{x}_0 &\sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0), \\
\mathbf{x}_k &= \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{w}_k, \quad & \mathbf{w}_k &\sim \mathcal{N}(0, \mathbf{Q}_k), \quad & k &\in \mathcal{K}, \\
\mathbf{y}_j &= \mathbf{H}^y_j \mathbf{x}_j + \mathbf{v}^y_j, \quad & \mathbf{v}^y_j &\sim \mathcal{N}(0, \mathbf{R}^y_j), \quad & j &\in \mathcal{J} \subseteq \mathcal{K}.
\end{aligned}
$$

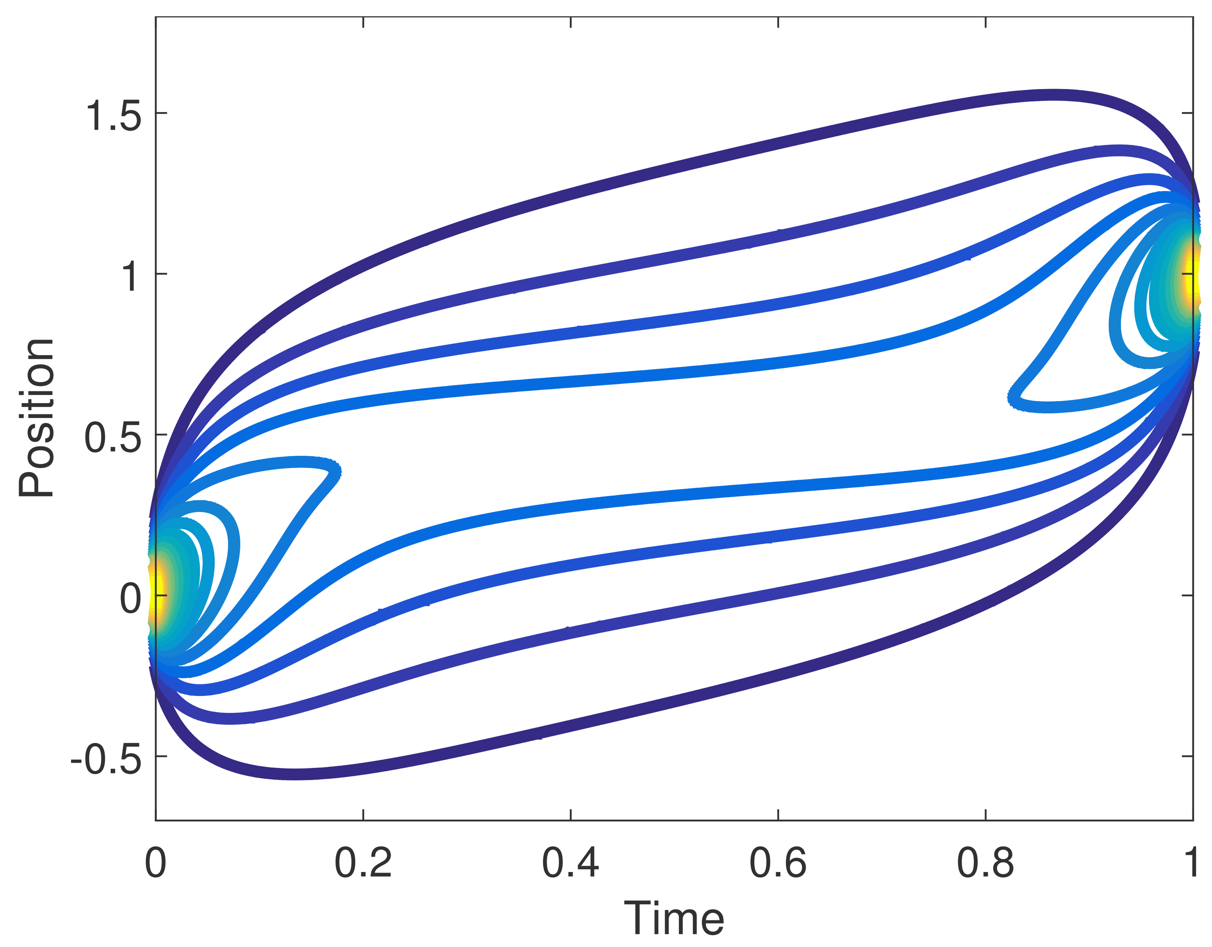
An observation with an uncertain timestamp $\tau \in \mathcal{K}$ is augmented to the model,

$$
\mathbf{z} = \mathbf{H}^z \mathbf{x}_\tau + \mathbf{v}^z, \qquad \mathbf{v}^z \sim \mathcal{N}(0, \mathbf{R}^z), \qquad \tau \sim p(\tau).
$$

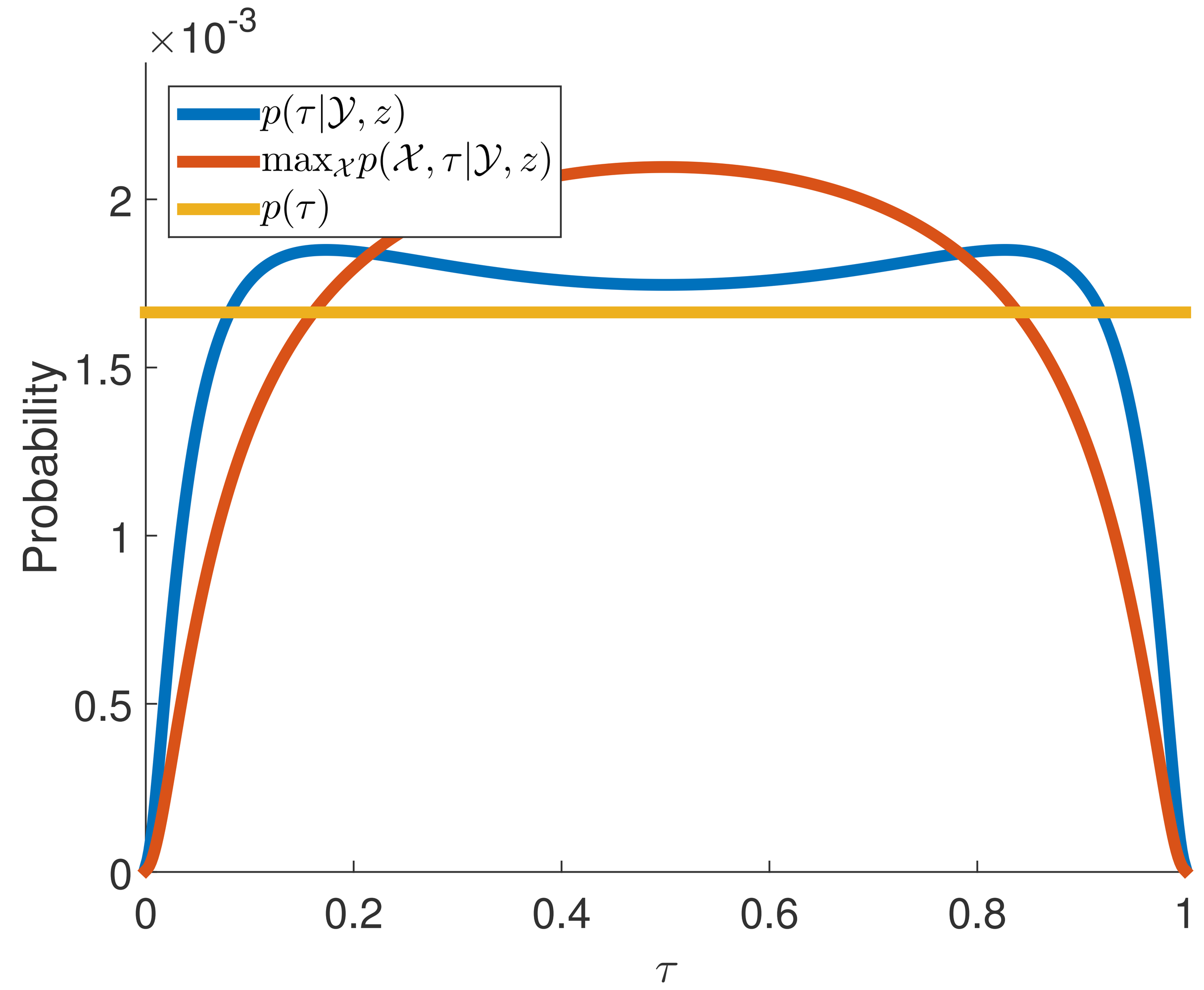
Simple Scenario

A simple one-dimensional scenario is considered for analysis. Two traditional measurements and one observation with an uncertain timestamp are available.

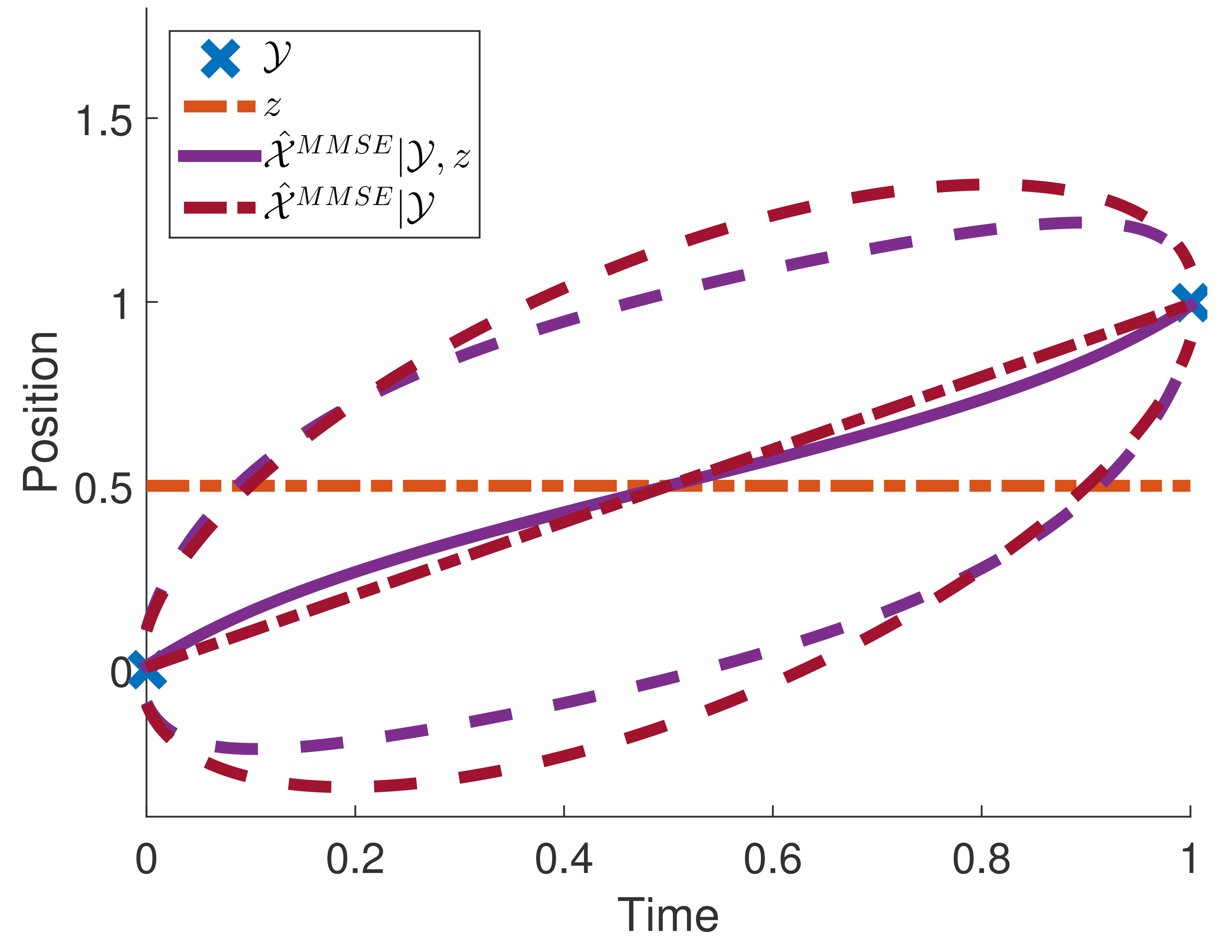
The posterior distribution of the states is a Gaussian mixture that is pushed towards the observed position.



The posterior distribution of the timestamp is not counter-intuitive.



The MMSE estimate of the states tends towards the observed position compared to the estimate without the observation.



Simple Scenario Model

A simple one-dimensional model is considered,

$$
\begin{aligned}
x_k &= x_{k-1} + w_k, \qquad & w_k &\sim \mathcal{N}(0, Q), \\
y_j &= x_j + v^y_j, \qquad & v^y_j &\sim \mathcal{N}(0, R^y), \\
z &= x_\tau + v^z, \qquad & v^z &\sim \mathcal{N}(0, R^z),
\end{aligned}
$$

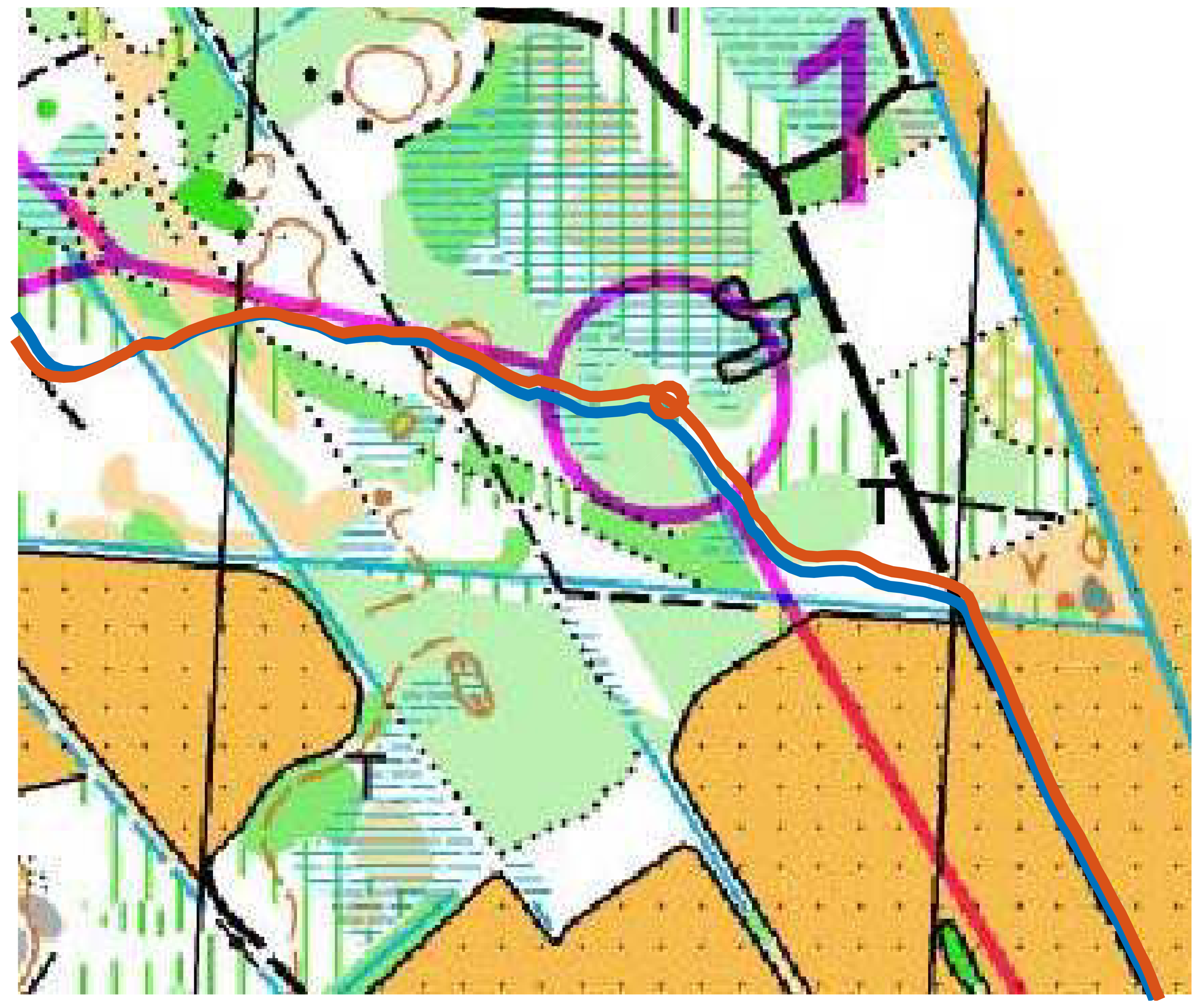
with $y_1 = 0$, $y_N = 1$, $z = 0.5$ and $p(\tau) \propto 1$.

Applications

Tracking of Animals: In addition to traditional measurements, such as from radars or cameras, a trace left by an animal can be seen as an accurate observation of the position with an uncertain timestamp.

Crime Scene Investigations: The place of the crime often is known accurately, but not the time, and witness statements often contain uncertainty in both time and place, while surveillance cameras are precise in time.

Sprint Orienteering: A control position in sprint orienteering can be used as an observation with an uncertain timestamp to improve the GPS position of the sprinter.



Conclusions & Future Work

- The estimate is improved by the observation with an uncertain timestamp.
- Computationally demanding even for simple systems.
- Consider multiple observations with uncertain timestamps.
- Consider nonlinear models.
- Derive posterior distributions in continuous time.

LINKÖPING UNIVERSITY
LINK-SIC

LINKÖPING UNIVERSITY
Division of Automatic Control