Is it possible to solve complex problems with simple tools?

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Background

In order to make control law design effective one needs to make the most of the available system information. This is very true for the case of the control law design of a modern fighter aircraft. The flight characteristics of this kind of system varies from stable to unstable, from linear to nonlinear, and the flight control system needs to deal with all combinations of these. Also, the process noise characteristics for atmospheric flight is colored, which adds to the system identification complexity.

This gives rise to some:

Challenges:

- Nonlinear system
- Closed-loop data
- Partially unknown disturbance characteristics

Engineering constraints:

- Accuracy
- Scalability
- User-independent system identification results

Theory

Flight dynamics can, in general, be described as a nonlinear system

$$x_{k+1} = F(x_k, u_k, w_k) \tag{1a}$$

$$y_k = H(x_k, u_k, v_k) \tag{1b}$$

where F describes the nonlinear dynamics of flight including the effect of the colored noise from atmospheric turbulence w_k, and h is the measurement equation disturbed by white measurement noise v_k. For the aircraft application the measurement is $y_k = x_k + v_k$.

A prediction error method (PEM) is used for the system identification:

$$\hat{x}_{k+1}(\theta) = F_m(\hat{x}_k(\theta), u_k, \theta) + K_k(\theta)\varepsilon_k(\theta) \tag{2a}$$

$$\hat{y}_k(\theta) = \hat{x}_k(\theta) \tag{2b}$$

$$\varepsilon_k(\theta) = y_k - \hat{y}_k(\theta) \tag{2c}$$

So, the question is ...

Can a simple parametrized observer (PO) stabilize this predictor?

In this approach the observer gain $K_k(\theta)$ is assumed to be constant and is added to the parameters to be identified:

$$\theta = \begin{bmatrix} \theta_f \\ \theta_K \end{bmatrix} \tag{3}$$

where θ_f are the parameters that appear in f and $\theta_K = \text{vec}(K_k)$ is a vector containing the observer gain parameters.

This is compared to another approach using the extended Kalman filter (EKF), where $K_k(\theta)$ is computed at each time step using a linearized model:

$$\begin{aligned} P^{xx}_{k|k-1}(\theta) &= A_k(\theta)\, P^{xx}_{k-1|k-1}(\theta)\, A_k^T(\theta) + Q \\ K_k(\theta) &= P^{xx}_{k|k-1}(\theta)\, C^T(\theta)\, [C(\theta)\, P^{xx}_{k|k-1}(\theta)\, C^T(\theta) + R]^{-1} \\ P^{xx}_{k|k}(\theta) &= (I - K_k(\theta)\, C(\theta))\, P^{xx}_{k|k-1}(\theta) \end{aligned} \tag{4}$$

where the predicted covariance matrix $P^{xx}_{k|k}(\theta)$ represents the uncertainty of the state prediction. Q and R are tuning parameters set by the user.

Example

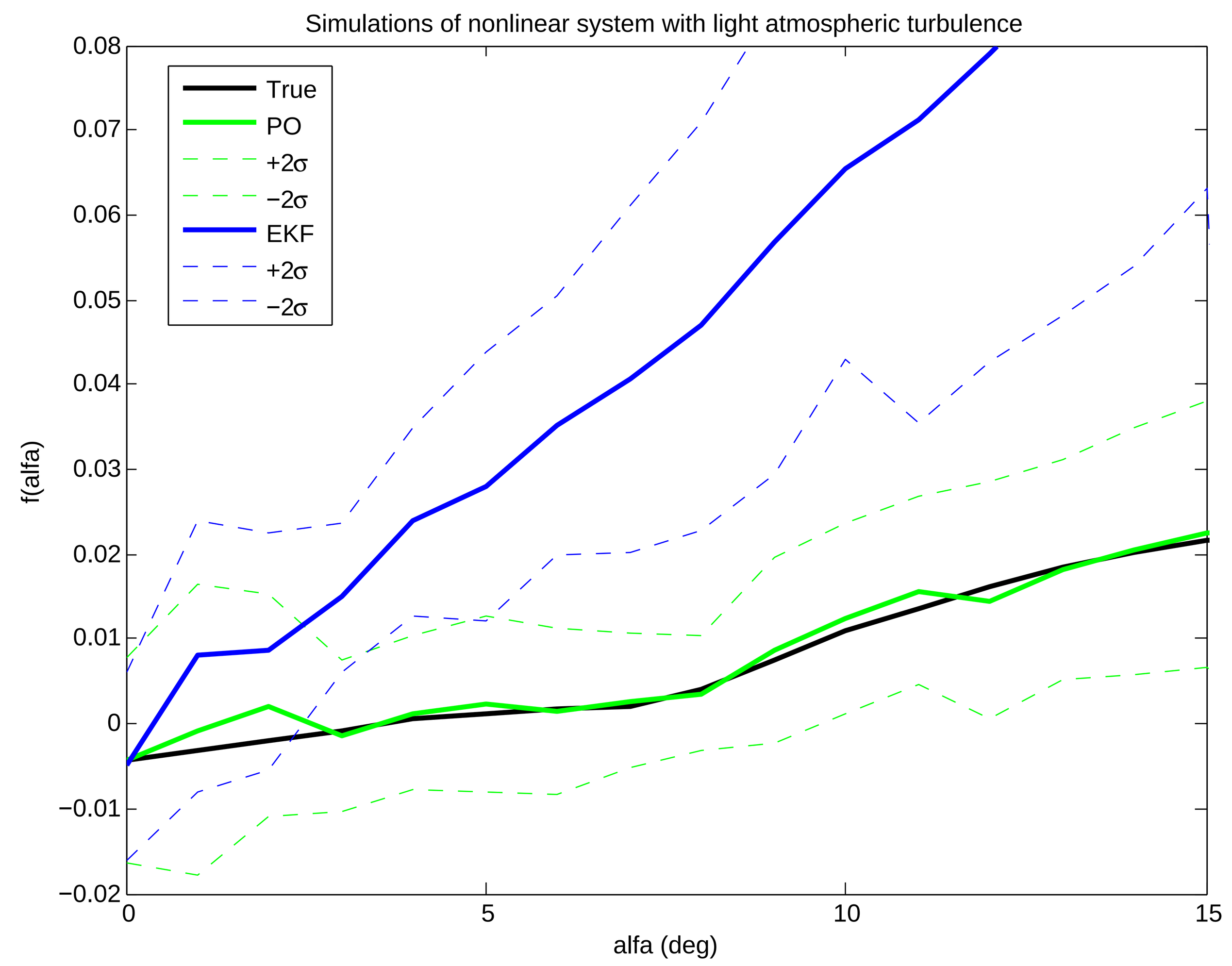
For this example the dynamic model in (2a) is given by

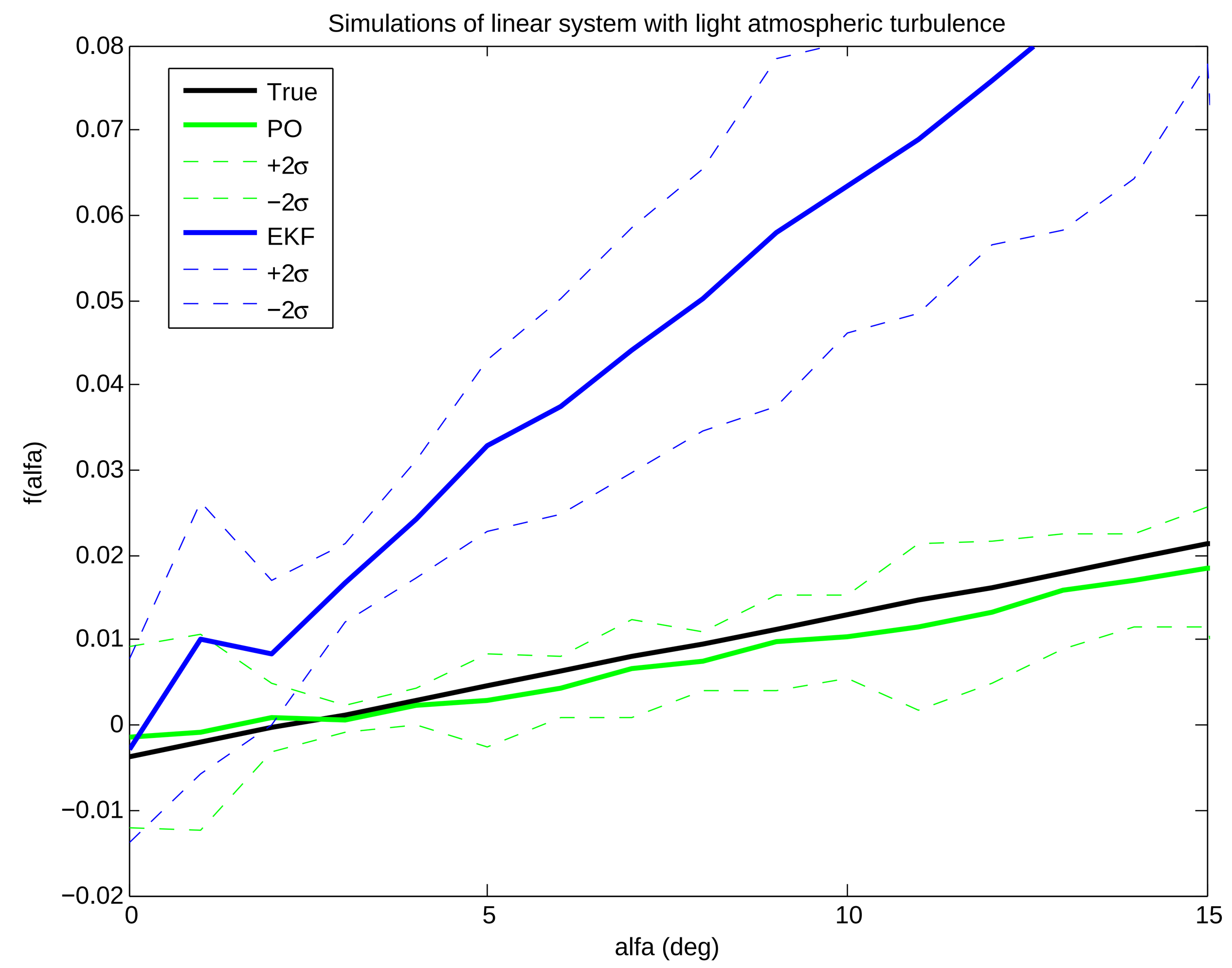
$$\begin{bmatrix} \hat{\alpha}_{k+1} \\ \hat{q}_{k+1} \end{bmatrix} = \begin{bmatrix} \theta_1\hat{\alpha}_k + \theta_2\hat{q}_k \\ T_s f(\theta_8, \ldots, \theta_{25}, \hat{\alpha}_k) + \theta_3\hat{q}_k \end{bmatrix} + \begin{bmatrix} \theta_4 & \theta_5 \\ \theta_6 & \theta_7 \end{bmatrix} \begin{bmatrix} \delta_{e\,k} \\ \delta_{c\,k} \end{bmatrix} \tag{5}$$

which has two states (angle-of-attack α and pitch-rate q), two inputs (elevator δ_e and canard δ_c) and 25 parameters to be estimated. The nonlinearity is represented by a piece-wise affine function of the angle-of-attack.

Results

Simulations have been performed with light turbulence to mimic a real flight test. In these the pitch stability of the aircraft is unstable and nonlinear as shown in the top figure. The lower figure shows a case where the system has been linearized, but still, a nonlinear predictor is used.





- The results indicate that the simple PO approach does a better job than the EKF approach in identifying the system under consideration, both in the linear and nonlinear case.
- There are no tuning parameters in the PO approach that the user has to set making the PO approach a easy-to-use engineering tool.

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