A Nonlinear Least-Squares Approach to the SLAM Problem

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Main idea

If a SLAM solution is considered as an initial point estimate it can be improved using (non)linear least-squares. This corresponds to fitting the measurement and process noise that minimises the noise variance for the smoothed estimate.

Summary

In this work we present a solution to the simultaneous localisation and mapping (SLAM) problem using a camera and inertial sensors. Our approach is based on the maximum a posteriori (MAP) estimate of the complete SLAM estimate. The resulting problem is posed in a nonlinear least-squares framework which we solve with the Gauss-Newton method. The proposed algorithm is evaluated on experimental data using a sensor unit mounted on an industrial robot.

Models

- The dynamic state parametrise full 6DOF and linear velocity with inertial sensors as inputs
- The camera is modeled as a standard pinhole camera
- Landmarks are considered to be stationary and they are parametrised using the unified inverse depth parametrisation:

$$\begin{bmatrix} X^e \\ Y^e \\ Z^e \end{bmatrix} = \begin{bmatrix} x^e \\ y^e \\ z^e \end{bmatrix} + \frac{1}{\rho^e} m(\varphi^e, \theta^e), \tag{1a}$$

$$m(\varphi^e, \theta^e) = \begin{bmatrix} \cos\varphi^e \sin\theta^e \\ \sin\varphi^e \sin\theta^e \\ \cos\theta^e \end{bmatrix}. \tag{1b}$$

Problem Formulation

The dynamic model and the measurements are on the following form

$$x_t = f(x_{t-1}, u_t) + \widetilde{w}_t, \tag{2a}$$
$$l_t = l_{t-1}, \tag{2b}$$
$$y_{t_k} = h(x_{t_k}, l_{t_k}) + e_{t_k}, \tag{2c}$$

where x_t and l_t are vehicle and landmark states, respectively, and the inertial measurements are modelled as inputs u_t. Assume that all the measurements and the inputs for $t = \{0 : N\}$ and $k = \{1 : K\}$ $(K \ll N)$ are available and the noise is i.i.d., then the joint probability density of (2) is

$$p(x_{0:N},\ l_N | y_{1:K},\ u_{1:N}) = p(x_0) \prod_{t=1}^{N} p_{\widetilde{w}_t}(x_t | x_{t-1}, u_t) \prod_{k=1}^{K} p_{e_{t_k}}(y_{t_k} | x_{t_k}, l_{t_k}). \tag{3}$$

Note that the map, l_N, is static and the estimate is given for the last time step only. The smoothed maximum a posteriori (MAP) estimate of $x_{0:N}$ and l_N is then

$$[x^*_{0:N}\ l^*_N] = \underset{x_{0:N},\ l_N}{\arg\max}\ p(x_{0:N},\ l_N | y_{1:K},\ u_{1:N}) \tag{4}$$

If the noise terms $\widetilde{w}_t$ and e_{t_k} are assumed to be Gaussian and white, i.e., $e_{t_k} \sim \mathcal{N}(0, R_{t_k})$ and $\widetilde{w}_t \sim \mathcal{N}(0, \widetilde{Q}_t)$, (4) can be posed as a nonlinear least-squares problem.

Nonlinear least-squares

$$[x^*_{0:N},\ l^*_N] = \underset{x_{0:N},\ l_N}{\arg\min} \sum_{t=1}^{N} ||x_t - f(x_{t-1}, u_t)||^2_{\widetilde{Q}_t^{-1}} + \sum_{k=1}^{K} ||y_{t_k} - h(x_{t_k}, l_{t_k})||^2_{R^{-1}_{t_k}} \tag{5}$$

Experimental Results

The nonlinear least-squares problem (5) is solved with the Gauss-Newton method where the initial estimate is given by an EKF SLAM solution.

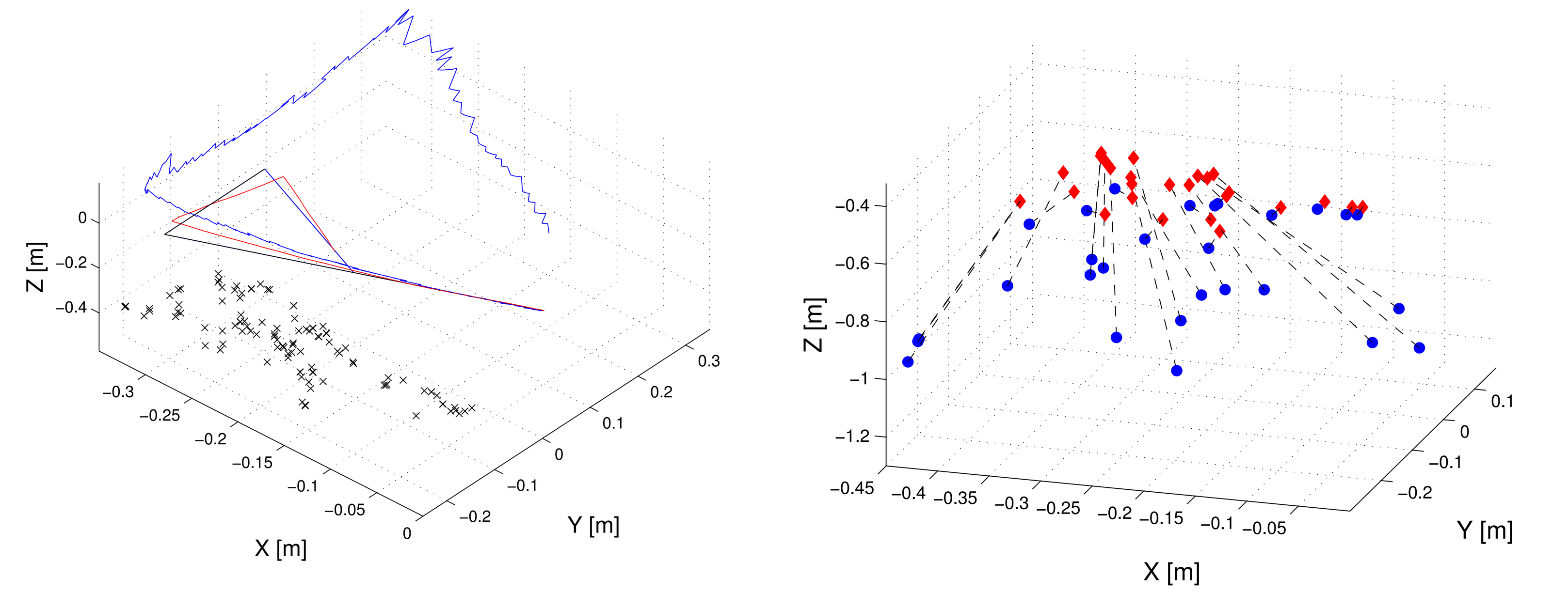

Figure 1: Left: The final nonlinear least-squares trajectory estimate in red, initial EKF trajectory in blue and ground truth trajectory in black. The black crosses are the smoothed landmark estimates. Right: The initial landmark estimates given by EKF in blue and the final nonlinear least-squares estimate in red.

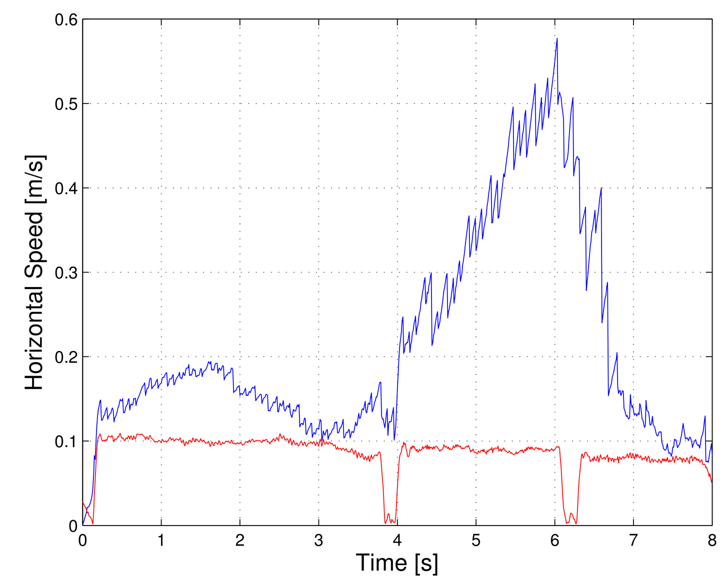

Figure 2: The smoothed horizontal speed of the camera in red and EKF in blue. The true speed is 0.1 m/s except for when the robot stops and changes direction, this happens at about 4 seconds and 6 seconds.

Future Work

- Estimate the initial trajectory on the fly using e.g. visual odometry
- Incremental constrained optimisation solution
- C++ implementation and evaluation on real flight data

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