Main idea
If a SLAM solution is considered as an initial point estimate it can be improved using (non)linear least-squares. This corresponds to fitting the measurement and process noise that minimises the noise variance for the smoothed estimate.

Summary
In this work we present a solution to the simultaneous localisation and mapping (SLAM) problem using a camera and inertial sensors. Our approach is based on the maximum a posterior (MAP) estimate of the complete SLAM estimate. The resulting problem is posed in a nonlinear least-squares framework which we solve with the Gauss-Newton method. The proposed algorithm is evaluated on experimental data using a sensor unit mounted on an industrial robot.

Problem Formulation
The dynamic model and the measurements are on the following form

\[
\begin{align*}
    x_t &= f(x_{t-1}, u_t) + \tilde{w}_t, \\
    l_t &= l_{t-1}, \\
    y_t &= h(x_t, l_t) + \epsilon_t,
\end{align*}
\]

where \(x_t\) and \(l_t\) are vehicle and landmark states, respectively, and the measurements and the inputs for \(t = \{0 : N\}\) and \(k = \{1 : K\}\) are available and the noise is i.i.d., then the joint probability density of (2) is

\[
p(x_{0:N}, l_N | y_{1:N}, u_{1:N}) = p(x_0) \prod_{t=1}^{N} p_{h_0}(x_t | x_{t-1}, u_t) \prod_{k=1}^{K} p_{h_k}(y_t | x_t, l_t).
\]

(3)

Note that the map, \(l_N\), is static and the estimate is given for the last time step only. The smoothed maximum a posteriori (MAP) estimate of \(x_{0:N}\) and \(l_N\) is then

\[
[x_{0:N}^*, l_N^*] = \text{arg max}_{x_{0:N}, l_N} p(x_{0:N}, l_N | y_{1:N}, u_{1:N})
\]

(4)

If the noise terms \(\tilde{w}_t\) and \(\epsilon_t\) are assumed to be Gaussian and white, i.e., \(\epsilon_t \sim \mathbf{N}(0, Q_t)\) and \(\tilde{w}_t \sim \mathbf{N}(0, R_t)\), (4) can be posed as a nonlinear least-squares problem.

\[
[x_{0:N}^*, l_N^*] = \text{arg min}_{x_{0:N}, l_N} \sum_{t=1}^{N} ||x_t - f(x_{t-1}, u_t)||_Q + \sum_{k=1}^{K} ||y_t - h(x_t, l_t)||_{R_k}^2
\]

(5)

Experimental Results
The nonlinear least-squares problem (5) is solved with the Gauss-Newton method where the initial estimate is given by an EKF SLAM solution.

Figure 1: Left: The final nonlinear least-squares trajectory estimate in red, initial EKF trajectory in blue and ground truth trajectory in black. The black crosses are the smoothed landmark estimates. Right: The initial landmark estimates given by EKF in blue and the final nonlinear least-squares estimate in red.

Future Work
- Estimate the initial trajectory on the fly using e.g. visual odometry
- Incremental constrained optimisation solution
- C++ implementation and evaluation on real flight data

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Models
- The dynamic state parametrise full 6DOF and linear velocity with inertial sensors as inputs
- The camera is modeled as a standard pinhole camera
- Landmarks are considered to be stationary and are parametrised using the unified inverse depth parametrisation:

\[
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \frac{1}{\theta'} m(\phi', \theta'),
\]

(1a)

\[
m(\phi', \theta') = \begin{bmatrix} \cos \phi' \sin \theta' \\ \sin \phi' \sin \theta' \\ \cos \theta' \end{bmatrix}.
\]

(1b)

Nonlinear least-squares

\[
[x_{0:N}^*, l_N^*] = \text{arg min}_{x_{0:N}, l_N} \sum_{t=1}^{N} ||x_t - f(x_{t-1}, u_t)||_Q + \sum_{k=1}^{K} ||y_t - h(x_t, l_t)||_{R_k}^2
\]

(5)