Estimation of MIMO Helicopter Models in a Partially Closed Loop Setting

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Main Idea

In some applications, identification experiments for MIMO systems are performed in a partially closed loop setting. From an identification point of view, the main question is whether such experiments provide enough information for the model parameters to be estimated. Just like in many other settings, the information matrix turns out to be useful.

Background

A number of interesting results about the information content of closed-loop experiments have been published recently. For example:

- *For the identification of a MIMO system based on closed-loop data collected with a linear time-invariant controller, it is not always necessary to excite all reference signals.*

  A.S. Bazanella, M. Gevers and L. Miskovic, "Closed-loop identification of MIMO systems: a new look at identifiability and experiment design", European Journal of Control, Vol. 16, No 3, pp. 228-239, May-June 2010.

- *The conditions for a full-rank information matrix in closed-loop identification is a quantifiable tradeoff between controller complexity and the degree of richness of the external excitation.*

  M. Gevers, A.S. Bazanella, X. Bombois and L. Miskovic, "Identification and the Information Matrix: How to Get Just Sufficiently Rich?", IEEE Transactions on Automatic Control, Vol. 54, No 12, pp. 2828-2840, December 2009.

With these inspiring, but somewhat surprising, results in mind, it seems natural that a careful analysis is needed also in the present setting.

Main Tool: The Information Matrix

Consider a MIMO system with m inputs and p outputs and a parameterized predictor model

$$\hat{y}(t|t-1,\theta) = W(q,\theta)z(t),$$

where $z(t) = \big(u(t)^T \; y(t)^T\big)^T$.

The information matrix:

$$I(\theta) = \sum_{k=1}^{p} \frac{1}{2\pi\lambda_k} \int_{-\pi}^{\pi} \frac{\partial W_k(e^{i\omega},\theta)}{\partial\theta} \Phi_z(e^{i\omega}) \frac{\partial W_k(e^{i\omega},\theta)}{\partial\theta}^H d\omega$$

(W_k = row k in W)

Estimation of Helicopter Dynamics

In some of the test flights performed during the development of an autonomous helicopter, the helicopter is controlled mainly by a pilot on the ground. However, in order to help the pilot, some control loops are typically closed also in this setup. For example, the vertical dynamics of a helicopter can be described by a second order model

The Skeldar helicopter

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Nv(t),\\ y(t) &= x(t) + w(t),\end{aligned}$$

where

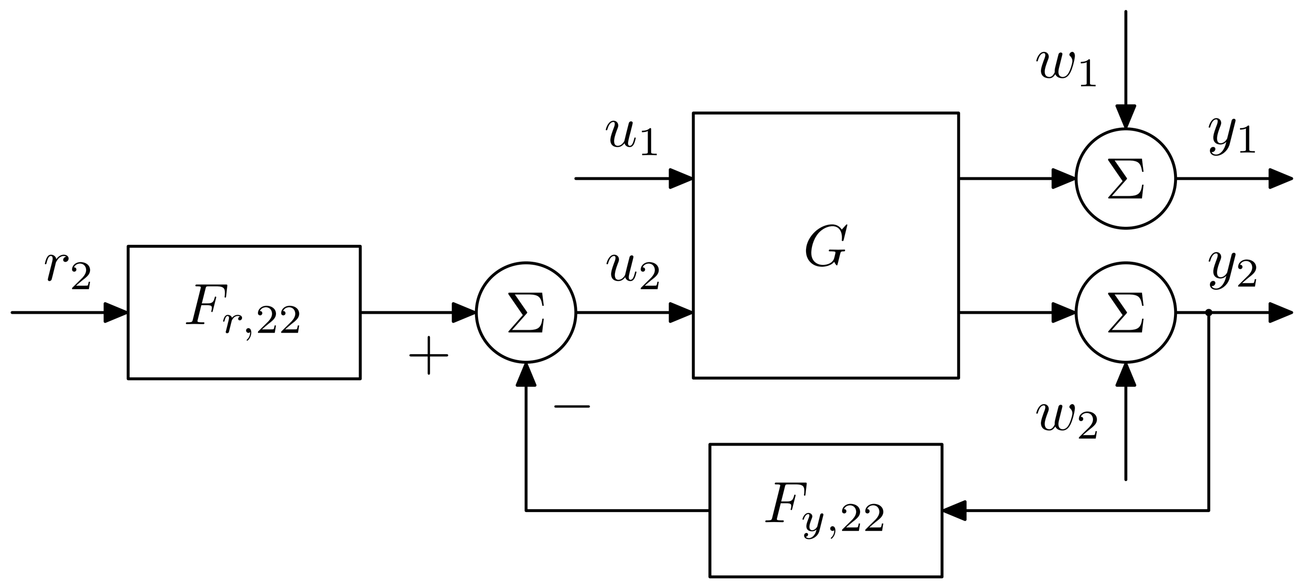
$$\begin{aligned} y_1(t) = x_1(t) &= \text{vertical velocity}\\ y_2(t) = x_2(t) &= \text{rotor angular velocity}\\ u_1(t) &= \text{collective pitch (pilot signal)}\\ u_2(t) &= \text{throttle signal (feedback control signal)}\end{aligned}$$

Numerical Example

Setup

System:

$$\begin{aligned} x(t+1) &= \begin{pmatrix} 0.57 & -0.17 \\ 0.63 & 0.45 \end{pmatrix} x(t) + \begin{pmatrix} 0.89 & -0.86 \\ 0.82 & 0.72 \end{pmatrix} u(t),\\ y(t) &= x(t) + w(t).\end{aligned}$$



Case 1

The analysis of $I(\theta_0)$ gives:

- $F_{y,22} = F_{r,22}$ = PI-controller that results in **real** closed-loop poles, $r_2 = 0$, u_1 and w_1 nonzero white noise, $w_2 \approx 0 \Rightarrow \lambda_{\min}(I(\theta_0)) \approx 0 \Leftrightarrow$ **Does not work!**
- $F_{y,22} = F_{r,22}$ = PI-controller that results in **complex** closed-loop poles, $r_2 = 0$, u_1 and w_1 nonzero white noise, $w_2 \approx 0 \Rightarrow I(\theta_0)$ better conditioned, **Seems to work rather well!**
- $F_{y,22} = F_{r,22}$ = PI-controller that results in real closed-loop poles, $r_2 = 0$, u_1, w_1 **and w_2** nonzero white noise $\Rightarrow I(\theta_0) > 0 \Leftrightarrow$ **OK!**

The results have been verified numerically in simulations.

Case 2

With an additional feedforward term from $u_1(t)$, i.e.,

$$u_2(t) = F_{r,22}(q)r_2(t) - F_{y,22}(q)y_2(t) + f_0 u_1(t),$$

with $f_0 = -B_{21}/B_{22}$, the analysis of $I(\theta_0)$ gives:

- The feedforward term **decreases** most variances compared to Case 1 when the regularity of $I(\theta_0)$ is obtained with a nonzero w_2. (Exceptions: B_{12} and B_{22})

Conclusions & Future Work

- Some of the obtained results seem difficult to figure out using only intuition. This shows that the information matrix is a very useful tool also for partially closed loop systems.
- Ideas for future work: A more detailed analysis of the conditions that result in $I(\theta_0) > 0$, system noise, low-pass filter in the feedback loop, colored reference and noise signals

Acknowledgments

This work has been performed in cooperation with Saab Aeronautics and Dr. Ola Härkegård within the VINNOVA Industry Excellence Center LINK-SIC.