Estimation of aerodynamic model parameters

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Background

Today's highly maneuverable fighter aircraft rely to a large extent on Model Based System Engineering (MBSE) when developing flight control systems (FCS). It is therefore highly desirable to have accurate models that span the whole flight envelope. The aerodynamic characteristics of the JAS 39 Gripen aircraft include regions that change from unstable to stable as well as from linear to nonlinear making the aerodynamic model complex. Therefore the engineer working with identification of aerodynamic characteristics based on flight test data need accurate tools. In order to ensure that the tests contain enough information an on-line tool working in real-time could be useful for surveying the amount of excitation in the collected data. After the test more advanced tools are needed to be able to work out the complexity of the model.

Frequency domain real-time method

A real-time frequency method has been studied. The method uses the fact that, for a linear system:

$$\begin{aligned} \dot{x} &= A\,x + B\,u + Nw \\ y &= C\,x + D\,u + v \end{aligned} \tag{1}$$

an input $u = A_u \cos(\omega\, t + \phi_u)$ gives an output $y = A_y \cos(\omega\, t + \phi_y)$, i.e., the frequency ω is unaffected by the system. In the frequency domain the parameter identification can be solved as a complex least squares regression:

$$\tilde{z} = \tilde{X}\,\theta + \tilde{e} \tag{2}$$

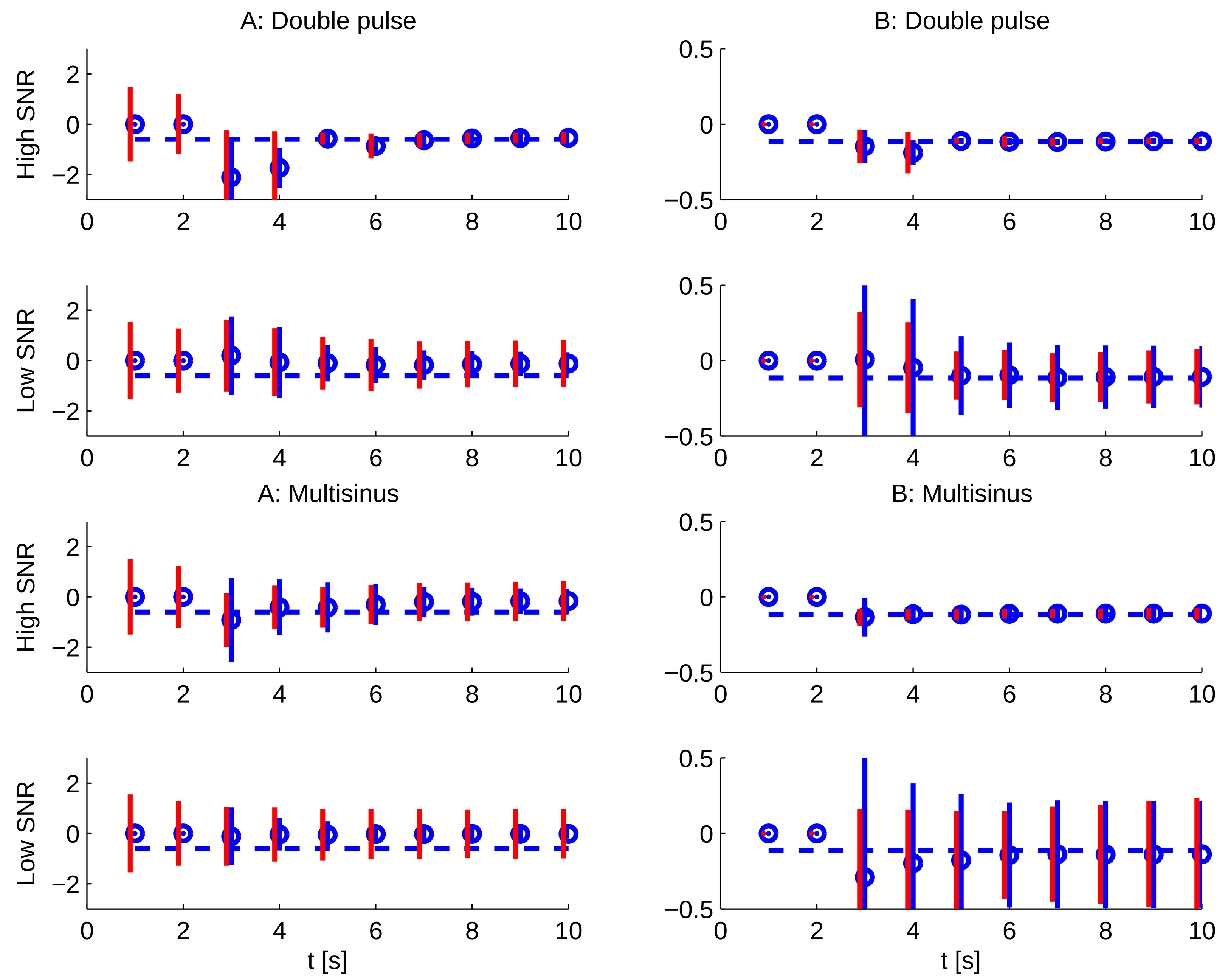
where the estimated parameters are given by

$$\hat{\theta} = (Re(\tilde{X}^{\dagger}\,\tilde{X}))^{-1} Re(\tilde{X}^{\dagger}\,\tilde{z}) \tag{3}$$

In this method the discrete Fourier transform is used recursively.

$$X_k(\omega) = \sum_{i=0}^{k} x(i)\, e^{-j\omega i \Delta t} = X_{k-1}(\omega) + x(k)\, e^{-j\omega k \Delta t} \tag{4}$$

That is, the transform at the current time is made up of the transform from the previous time step plus the contribution from the current time step. This means that all information from earlier time steps is stored in $X_{k-1}(\omega)$, much like the state in a state space model.



The figure shows results from a 1-D test case where the properties of the 95% confidence interval estimation have been studied. The blue bars come from Monte Carlo simulations and the red bars show the estimates produced by the method using the same data.

Nonlinear estimation methods

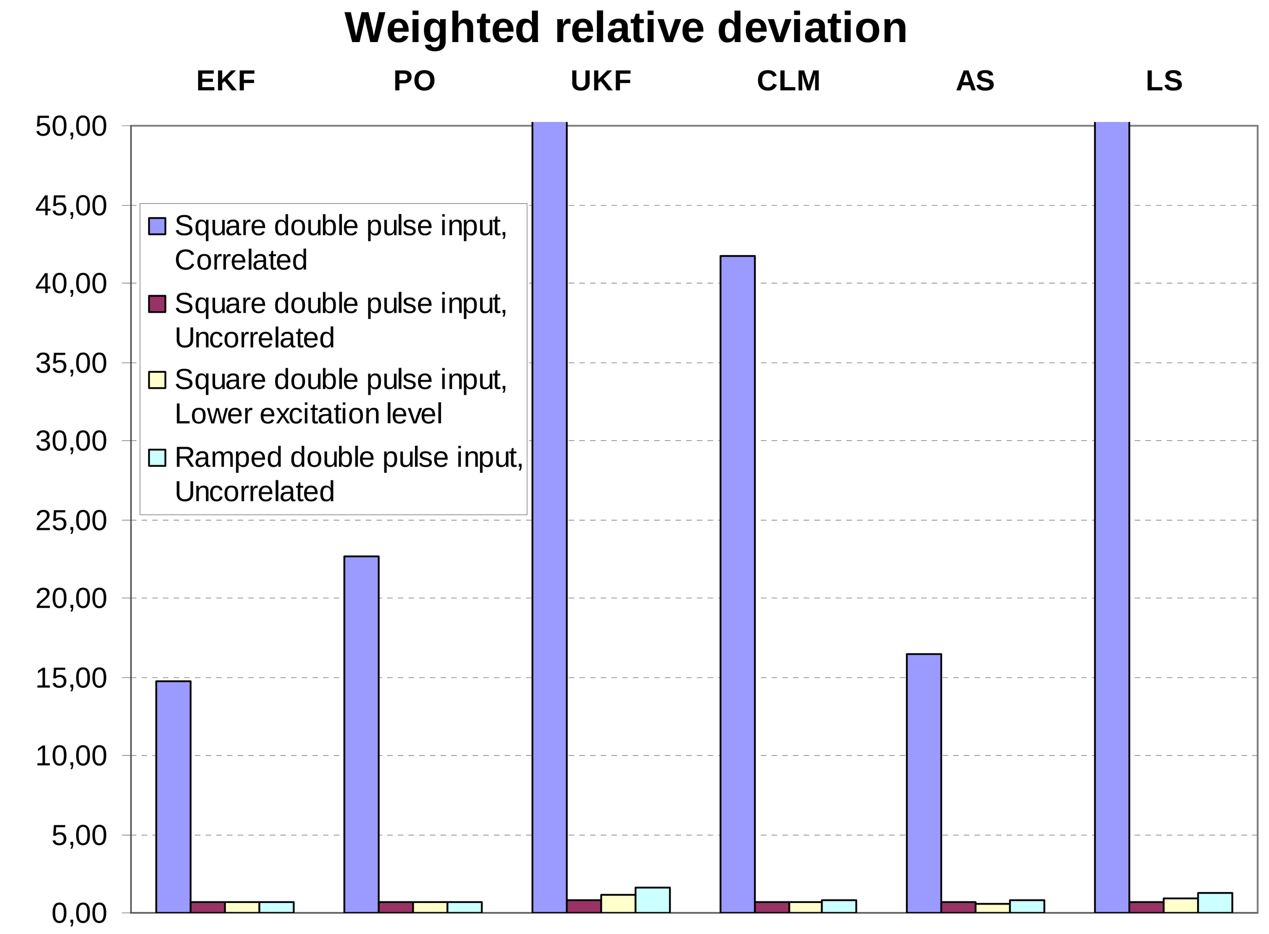
Nonlinear systems can be described as:

$$\begin{aligned} x_{t+1} &= f(x_t, u_t) + w_t \\ y_t &= h(x_t) + v_t \end{aligned} \tag{5}$$

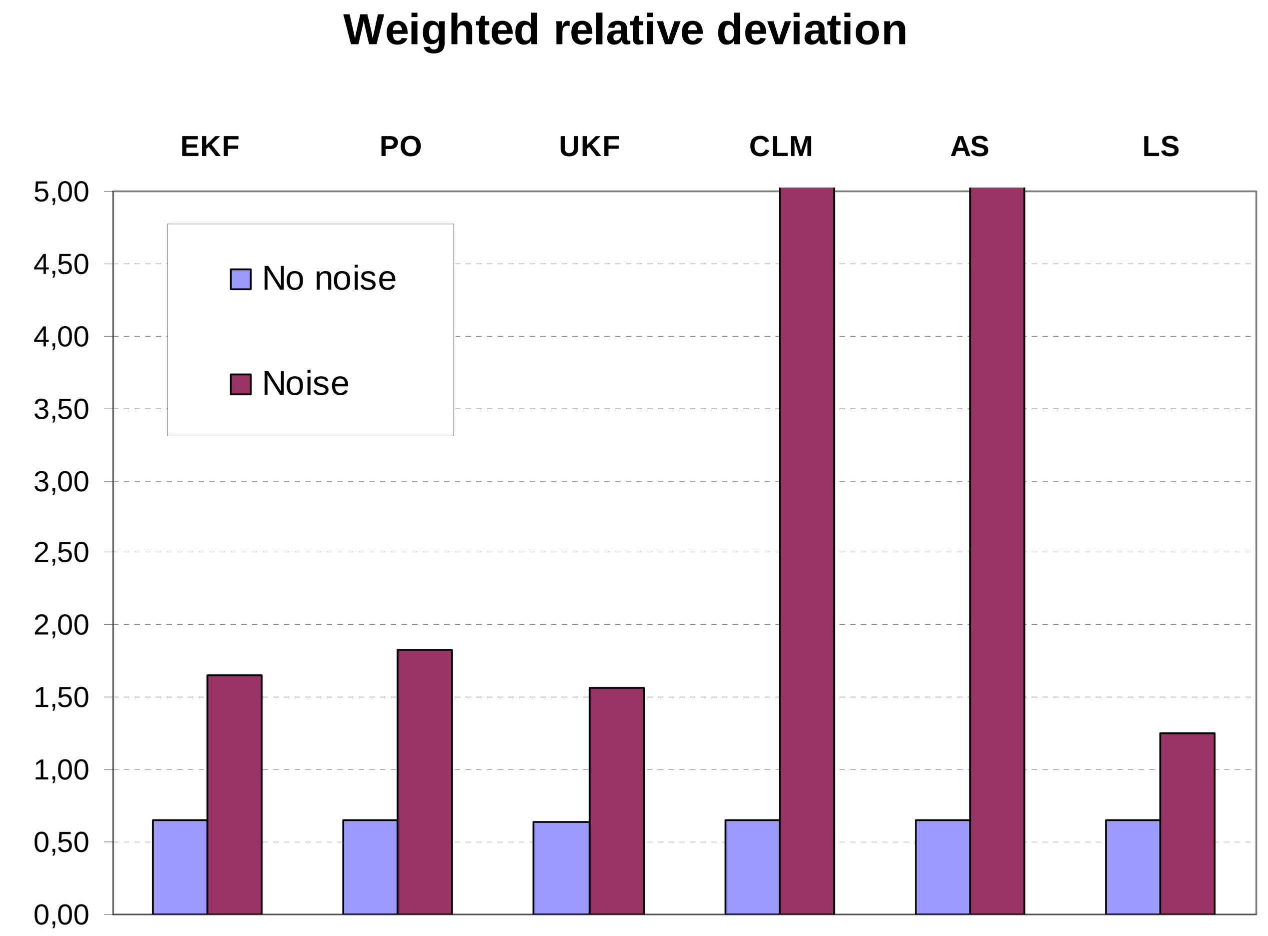
For six different methods:

1. Parameterized Observer (PO): $\hat{x}_{t+1}(\theta) = f(\hat{x}_t(\theta), u_t) + K_t^{PO}(\theta)\epsilon_t(\theta)$
2. Extended Kalman Filter (EKF): $\hat{x}_{t+1}(\theta) = f(\hat{x}_t(\theta), u_t) + K_t^{EKF}\epsilon_t(\theta)$
3. Unscented Kalman Filter (UKF): $\hat{x}_{t+1}(\theta) = \sum_i W_i f(\sigma_i, u_t)$
4. Constr. Levenberg-Marquardt (CLM): $f(\hat{x}_t(\theta), u_t) - \hat{x}_{t+1}(\theta) = 0$
5. Augmented System (AS): $\hat{\bar{x}}_t(\theta) = [\hat{x}_t(\theta)^T\ \theta^T]^T$
6. Least Squares (LS): $\hat{\theta} = [g(y_t, u_t)] \backslash [y_{t+1}]$

the linear sensitivities due to different inputs and noises have been studied for a simple test case. Some results are shown in the figures.



As can be seen the UKF, CLM and LS methods work very poorly with correlated inputs. Breaking up the correlation between the inputs gives much better estimates for all methods. Also UKF and LS are a bit more sensitive to the input shape.



The noise affects all methods, but the CLM and AS are by far the most sensitive.

Conclusions

- The real-time method seems promising. It is a linear method so care has to be taken how it is used.
- There are several methods for estimation of nonlinear systems. The EKF seems to be the most promising one so far.