A framework for analysis of observer-based ILC
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Background

Main points:
- Controlled variable not measured variable.
- Framework for ILC algorithm combined with procedure generating estimate of the controlled variable.
- Expression for asymptotic error of controlled variable when ILC algorithm based on estimate of controlled variable has converged.

System description

System $T$:  
- Time-invariant
- Iteration-invariant
- Linear
- Discrete
- Reference $r(t)$
- ILC input signal $u_k(t)$
- Measured variable $y_k(t)$
- Controlled variable $z_k(t)$

True system description at iteration $k$

$y_k(t) = T_{y_k}^0(q)r(t) + T_{y_k}^u(q)u_k(t)$
$z_k(t) = T_{z_k}^0(q)r(t) + T_{z_k}^u(q)u_k(t)$

$T_{y_k}^0$, $T_{y_k}^u$, $T_{z_k}^0$, $T_{z_k}^u$ stable and causal.

- System and measurement disturbances not included for simplicity.

Examples of $T$:  
- Open-loop system
- Closed-loop system

Estimation of controlled variable

The following parameterisation proposed: $F_r$, $F_u$ and $F_y$ assumed stable.

$\dot{z}_k(t) = F_r(q)r(t) + F_u(q)u_k(t) + F_y(q)y_k(t)$

Example 1 (Model of direct relation between $y_k$ and $z_k$).

There are situations with explicit relation $z_k(t) = T_{z_k}^0(q)y_k(t)$. By using a nominal model, it implies

$\dot{z}_k(t) = T_{z_k}^0(q)y_k(t)$

Example 2 (Observer). Based on nominal model of the state-space system. Linear observer formed as

$\dot{\hat{x}}(t + t_s) = A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t))$
$\dot{\hat{z}}_k(t) = M\hat{x}(t)$

Estimate of the controlled variable as

$\dot{\hat{z}}_k(t) = F_{\hat{z}}(q)\hat{u}_k(t) + F_{\hat{z}}y_k(t)$

From the closed-loop system, the filters in the parameterisation is

$\dot{\hat{z}}_k(t) = F_{\hat{z}}(q)(F(q)u_k(t) + F(q)r(t) - F(q)y_k(t)) + F_{\hat{z}}y_k(t)$

$F(q)r(t) + F(q)u_k(t) + F_{\hat{z}}y_k(t)$

ICL algorithm

Update equation for an ILC algorithm in transfer-operator form

$u_{k+1}(t) = Q_l(u_k(t) + L_ek_4(t))
\epsilon_4(t) = r(t) - \hat{z}_k(t)$

- Possibly non-causal linear filters $Q$ and $L$.
- System performance evaluated using $\epsilon_4(t) = r(t) - \hat{z}_k(t)$.

Results

Summary of main equations in matrix form:

The system given by

$y_k = T_{y_k}^0r + T_{y_k}^u u_k$
$z_k = T_{z_k}^0r + T_{z_k}^u u_k$

is controlled using the ILC input signal $u_k$, updated by

$u_{k+1} = Q_1(u_k + L_4\epsilon_4)$
$\epsilon_4 = r - \hat{z}_k$

using the estimate

$\hat{z}_k = F_r r + F_u u_k + F_y y_k$

Theorem 1 (Stability). Consider the ILC system equation

$u_{k+1} = Q_1(I - L(F_u + F_y T_{y_k}^0))u_k + Q_1L(I - (F_r + F_y T_{y_k}^0))r$

This system is stable iff $\rho(Q(I - L(F_u + F_y T_{y_k}^0))) < 1$, where $\rho(\cdot)$ is the spectral radius of the matrix.

Lemma 2 (Asymptotic behaviour). Assume input $u_k$ in the ILC system equation converges to a bounded signal as $k \to \infty$. The limit is

$u_\infty = \left(I - Q_1(I - L(F_u + F_y T_{y_k}^0))\right)^{-1}Q_1L(I - (F_r + F_y T_{y_k}^0))r$

The asymptotic error $r - z_\infty$ is

$e_\infty = \left(I - T_{z_k}^0F_{\hat{z}}T_{y_k}^0(I - Q_1(I - L(F_u + F_y T_{y_k}^0)))^{-1}Q_1L(I - (F_r + F_y T_{y_k}^0))\right)r$

Possible uses of expressions

Analyse influence of performance with respect to
- Design of feedback controller.
- Choice of estimator filters.
- Design of ILC algorithm.
- Model uncertainties and disturbances.